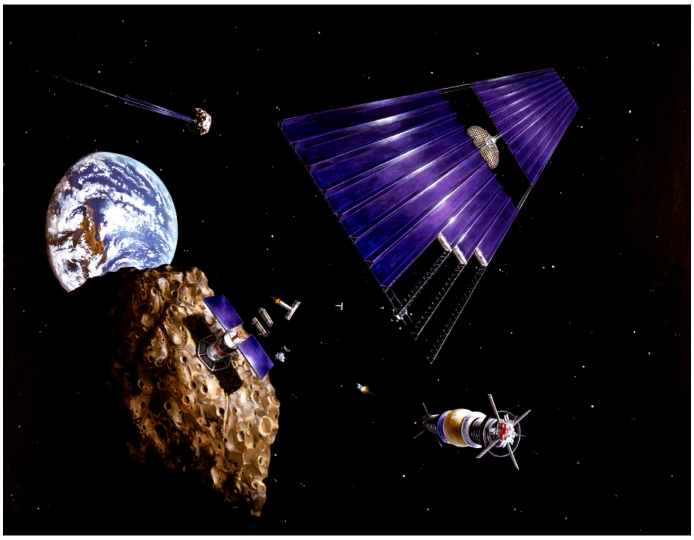
Problem Description for the
1st ASTROdynamics optiMIzation competitioN at
UC3M (ASTROMIN)



1 Introduction

This is the first competition on Astrodynamics and Optimization at UC3m. It is intended to pose a challenge in this field to engineering students in their Bachelor or Master degree. Competitors will count on available tools to develop their solution. In particular, the General Mission Analysis Tool (GMAT) developed by NASA is used to produce the final mission profile. This year the topic of the competition is Asteroid Mining. In the last few years, exploitation of asteroid resources has raised the interest of entrepreneurs and states, in a search for new sources of raw materials. Nowadays, there are companies and countries enlisted in the venture of developing the means to make asteroid mining possible. The challenge is great in many different aspects. One of them is related to the cost of going to the asteroid and bring the material back to Earth. This competition is in the line to assess the feasibility of asteroid mining from this point of view.

Two types of mission are defined, with two different objectives. The first one is to visit the maximum number of asteroids, whereas the second one is to bring back to Earth the maximum amount of asteroid material. Teams of two students can address either (or both) of the mission design challenge. Questions and doubts on the problem will be channeled through `piazza.com` [1]. Students are encouraged to solve competitors' doubts. A bonus will be awarded to the student with more (correct) answers.

This competition is inspired and based in the GTOC (Global Trajectory Optimisation Contest), particularly in its second edition (problem proposed by Anastassios Petropoulos, check original document `here`https://sophia.estec.esa.int/gtoc_portal/wp-content/uploads/2012/11/gtoc2_problem.pdf).

2 Problem Description

2.1 Summary

Asteroid mining is being considered as a profitable option by companies all over the world. They focus on the resources that are possibly available in asteroids. Nevertheless, arriving at an asteroid and coming back to Earth with minerals comes at a cost. The competition of this year is intended to assess the cost of bringing asteroid material to Earth orbit. In order to do so, we plan two missions. The aim of the first one is the physical characterisation of the larger possible number of asteroids for a given propellant and spacecraft masses. The goal of the second is to arrive to a specific asteroid and bring back to Earth the larger possible amount of material for a given propellant and spacecraft masses.

The first problem is a multiple asteroid rendezvous. A trajectory must be designed for a spacecraft which launches from Earth and subsequently performs a rendezvous with one asteroid from the database. Maximisation of the ratio of the number of visited asteroids to flight time is sought.

The second problem is a single asteroid rendezvous and a comeback trajectory. A trajectory must be designed for a spacecraft which launched from Earth, perform a rendezvous with one asteroid of the database and came back to Earth with an additional mass (accounting for the asteroid mining material). Maximisation of the ratio of the asteroid mining material to flight time is sought.

2.2 The mission and engineering parameters

The spacecraft is to be launched from the Earth, with a hyperbolic excess velocity (v_1) of up to 3.5 km/s and of unconstrained direction. The year of launch must lie in the range 2018 to 2038, inclusive. After launch, the spacecraft must rendezvous with one asteroid from the database. Choosing an asteroid from the database is part of the optimisation process. The order in which the asteroids are visited is immaterial. A stay time of at least 90 days is required at each asteroid. For the second problem, the initial mass of the trip back to Earth must take into account the amount of asteroid material m_{mined}. The flight time, t_f, measured from launch up to the point of rendezvous with the last asteroid, must not exceed 20 years. Gravity-assists are not permitted. Objective of the optimisation is to maximise the quantities:

$$J_1 = N_{\text{asteroid}}/t_f$$

[1] Piazza is "is a free online gathering place where students can ask, answer, and explore 24/7, under the guidance of their instructors."

Table 1: Keplerian orbit elements of the Earth, J2000 heliocentric ecliptic reference frame

Semimajor axis	a (AU)	0.999988049532578
Eccentricity	e	1.671681163160e-02
Inclination	i (deg.)	0.8854353079654e-03
RAAN	Ω (deg.)	175.40647696473
Arg. peri.	ω (deg.)	287.61577546182
Mean anomaly at epoch	M_0 (deg.)	257.60683707535
Epoch	t_0 (MJD)	54000

$$J_2 = m_{\text{mined}}/t_f$$

where J_1 is the objective of the first mission, J_2 is the objective of the second mission, andN_{asteroid} is the number of visited asteroids. The spacecraft has a fixed initial mass of 2500 kg (it does not change with launch v_1). The initial mass includes 2000 kg of available propellant. The propulsion is by means of a chemical thruster which can be turned on or off at will, has a constant specific impulse of 600 s, and has no constraint on the maximum thrust level. There is no constraint on the thrust direction, neither.

3 Dynamical models

The Earth and asteroids are assumed to follow Keplerian (conic) orbits around the Sun, as specified below. The only forces acting on the spacecraft is the Sun's gravity. Manoeuvres are modelled as instantaneous changes in velocity without a change in position. The file asteroid_Data.csv is an ASCII file giving the asteroid ephemeris in terms of Keplerian orbit elements and other physical data. The columns in the file are as follows: 1) A unique ID number of the asteroid; 2) semimajor axis in AU; 3) eccentricity; 4) inclination in degrees; 5) longitude of the ascending node in degrees; 6) argument of periapsis in degrees; 7) mean anomaly in degrees; 8) epoch, in modified Julian date, at which the mean anomaly is given; 9) the name of the asteroid. The orbit elements are expressed in the J2000 heliocentric ecliptic frame. The asteroids are listed by ID number. [The elements are taken from the public, small-body database maintained by JPL and accessible at `http://ssd.jpl.nasa.gov.`] The Earth's orbit elements are to be taken from Table 1. Other required constants are shown in Table 2.

4 Solution Format

Each team should return its best solution by email to manuel.sanjurjo@uc3m.es on or before 20 May 2017. Two files must be returned. The first file should contain:

1. a brief description of the methods used,
2. a summary of the best trajectory found (at least: IDs and group numbers of the asteroids visited, launch date, launch v_1, arrival and departure dates

Table 2: Table 2: Other constants and conversions

Gravitational parameter of the Sun	μ (km^3/s^2)	1.32712440018e11
Astronomical Unit	AU (km)	1.49597870691e+08
Standard acceleration due to gravity	g (m/s^2)	9.80665
Day	(s)	86400
Year	(days)	365.25
00:00 01 January 2018	(MJD)	58119.5
24:00 31 December 2038	(MJD)	65788.5

at the asteroids, spacecraft mass at the asteroids, total flight time, and value of the objective function in units of kg/year or 1/year),

3. a visual representation of the trajectory, such as a projection of the trajectory onto the ecliptic plane. The file should preferably be in Portable Document Format (PDF) or PostScript (PS) format; Microsoft Word format should also be acceptable.

The second file, which will be used to verify the solution returned, will be a GMAT mission file.

Appendix

As an aid for the students in this competition, this appendix provides a set of equations describing the dynamics of this problem along with other background information.

Nomenclature

Orbit elements and related quantities

a semimajor axis,	km
e eccentricity	
i inclination,	rad
Ω right ascension of the ascending node (RAAN),	rad
ω argument of periapsis,	rad
M mean anomaly,	rad
θ true anomaly,	rad
E eccentric anomaly,	rad
r radius from the Sun,	km
γ the flight path angle,	rad
μ Gravitational parameter of the Sun,	km^3/s^2

Cartesian position and velocity

x, y, z	the cartesian position coordinates of an orbiting body w.r.t Sun
$\vec{\mathbf{x}}$	vector of position coordinates, x, y, z
v_x, v_y, v_z	the cartesian velocity components of an orbiting body w.r.t Sun expressed in an inertial reference frame
$\vec{\mathbf{v}}$	vector of velocity components, v_x, v_y, v_z

Other quantities

t	time, s
t_f	flight time, measured from launch to beginning of rendezvous with fourth asteroid, years
m	spacecraft mass, kg
I_{sp}	specific impulse, s
T	thrust of propulsion system, N
g	standard acceleration due to gravity at Earth's surface, m/s^2
v_∞	hyperbolic excess velocity, km/s

Subscripts and superscripts

$()_f$	value of quantity measured at beginning of rendezvous with fourth asteroid
$()_0$	value of quantity at some given instant
$\dot{()}$	time derivative of quantity

Dynamics and conversions between elements

The motion of the Earth and asteroids around Sun is governed by these equations:

$$\begin{aligned} \ddot{x} + \mu\frac{x}{r^3} &= 0 \\ \ddot{y} + \mu\frac{y}{r^3} &= 0 \\ \ddot{z} + \mu\frac{z}{r^3} &= 0 \end{aligned}$$

where

$$r = \sqrt{x^2 + y^2 + z^2} = \frac{a(1-e^2)}{1 + e\cos\theta} \tag{1}$$

The motion of the spacecraft around the Sun is governed by the same formulas. The mass of propellant due to the rocket manoeuvres can be computed using Tsiolkovsky equation.

$$m_{\text{prop}} = m_{\text{ini}}\left(1 - \exp\left(-\frac{\Delta V}{gI_{sp}}\right)\right),$$

where m_{ini} is the mass before the manoeuvre and m_{prop} is the mass of propellant used for the manoeuvre.

Conversion from orbit elements to cartesian quantities is as follows:

$$\begin{aligned} x &= r\left[\cos(\theta+\omega)\cos\Omega - \sin(\theta+\omega)\cos i \sin\Omega\right] \\ y &= r\left[\cos(\theta+\omega)\sin\Omega + \sin(\theta+\omega)\cos i\cos\Omega\right] \\ z &= r\left[\sin(\theta+\omega)\sin i\right] \\ v_x &= v\left[-\sin(\theta+\omega-\gamma)\cos\Omega - \cos(\theta+\omega-\gamma)\cos i\sin\Omega\right] \\ v_y &= v\left[-\sin(\theta+\omega-\gamma)\sin\Omega - \cos(\theta+\omega-\gamma)\cos i\cos\Omega\right] \\ v_z &= v\left[\cos(\theta+\omega-\gamma)\sin i\right] \end{aligned}$$

where the velocity v is

$$v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$

, the flight path angle is obtained from

$$\tan\gamma = \frac{e\sin\theta}{1+e\cos\theta}$$

, the true anomaly is related to the eccentric anomaly by

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}$$

, the eccentric anomaly is related to the mean anomaly by Kepler's equation,

$$M = E - e\sin E$$

, and the mean anomaly is related to time and the initial mean anomaly by

$$M - M_0 = \sqrt{\frac{\mu}{a^3}}(t - t_0)$$

Thus, based on the provided ephemeris data, the cartesian positions and velocities of the Earth and asteroids may be computed as a function of time with only the minor nuisance of having to solve Kepler's equation for E by some iterative procedure. (That is, for the Earth, the asteroids, and a spacecraft, the equations of motion do not need to be numerically integrated to find position and velocity at some given time.)

4.1 Mathematical definition of launch and rendezvous

Given a launch date, t_L, and a launch hyperbolic excess velocity, $\vec{\mathbf{v}}_{\infty L}$, the initial position and velocity of the spacecraft, $\vec{\mathbf{x}}_s$ and $\vec{\mathbf{v}}_s$, are then given by

$$\vec{\mathbf{x}}_s(t_L) = \vec{\mathbf{x}}_E(t_L), \quad \vec{\mathbf{v}}_s(t_L) = \vec{\mathbf{v}}_E(t_L) + \vec{\mathbf{v}}_{\infty L}$$

where $\vec{\mathbf{x}}_E(t_L)$ and $\vec{\mathbf{v}}_E(t_L)$ are the position and velocity of the Earth at time t_L, and

$$|\vec{\mathbf{v}}_{\infty L}| \leq \ 3.5\,\mathrm{km/s}$$

.

As mentioned earlier, the effect of Earth's gravity on the spacecraft is to be ignored. (This approximation of the real physics of the problem is very good for heliocentric trajectories.) The mass of the spacecraft at launch is $m(t_L) = 2500$ kg. Asteroid rendezvous occurs at a time t_R when the spacecraft matches the position and velocity of the asteroid:

$$\vec{\mathbf{x}}_s(t_R) = \vec{\mathbf{x}}_A(t_R), \quad \vec{\mathbf{v}}_s(t_R) = \vec{\mathbf{v}}_A(t_R)$$

.

4.2 Glossary

gravity assist	A hyperbolic flyby of a [massive] body for purposes of achieving a desirable course change.
Modified Julian Date (MJD)	Has units of days and is defined as MJD=(Julian Date - 2400000.5), where the Julian Date is simply the number of days past some defined point in the past.
rendezvous	Meeting a body such as an asteroid by matching its position and velocity. The body is treated as a massless, moving point in space.
stay-time	A period of time during which the spacecraft remains in a state of rendezvous with a body.